

Isoperimetric inequalities

Prof. Anoop TV, IIT Madras

The classical isoperimetric problem asks: among all plane regions with a fixed perimeter, which one encloses the largest area? The isoperimetric inequality answers this question quantitatively and shows the circle (and in higher dimensions, the sphere) is uniquely optimal. In three dimensions, this answers why a soap bubble tends to become spherical. In the second half, we turn to the spectral analogue, the Faber–Krahn inequality: among all domains with fixed volume, the ball uniquely minimizes the first Dirichlet eigenvalue of the Laplacian (so, in the “drum” problem, the round drum has the lowest fundamental tone among drums of the same area/volume). We will sketch how symmetrization ideas link the geometric inequality to this eigenvalue minimization principle. We also discuss this question for the class of domains with holes.