Faculty Development Programme on Fundamentals of Mathematics and Pedagogy

Overall Structure of the FDP

The Faculty Development Programme is planned for a duration of five days and will have two broad components:

1. **Mathematics Component:** Lectures on Real Analysis and Linear Algebra.
2. **Pedagogy Component:** Lectures and activities focusing on the teaching and learning of undergraduate mathematics.

Each day consists of two major sessions. The first session, consisting of two lectures, will focus on mathematical content, while the second session, consisting of one lecture, will focus on pedagogy from the perspective of undergraduate mathematics teaching. Finally, we will have a tutorial/discussion session at the end of each day.

The mathematical component consists of ten one-hour sessions. The lectures will be based on the fundamental topics of mathematics from Linear algebra and real analysis.

The pedagogy component consists of five one-hour and fifteen-minute sessions. The emphasis will be on issues that arise specifically in teaching mathematics at the undergraduate level rather than on general educational theory.

Overall Structure of the Mathematical Component

The mathematical component of the Faculty Development Programme is planned as a sequence of ten one-hour sessions spread over five days.

The lectures will focus on fundamental topics from two core areas of undergraduate mathematics:

1. **Linear Algebra**
2. **Real Analysis**

The objective is not merely to review the standard syllabus, but to revisit fundamental concepts from a mathematical and pedagogical perspective. Particular emphasis will be placed on:

- conceptual understanding;

- connections between different topics;
- examples and counterexamples;
- common difficulties faced by undergraduate students;
- problem-solving techniques;
- different approaches to introducing and explaining fundamental ideas.

Part I: Linear Algebra

The Linear Algebra component will consist of five one-hour lectures covering the fundamental ideas normally encountered in a first undergraduate course.

Session 1: Systems of Linear Equations and Gaussian Elimination

Systems of Linear Equations and Gaussian Elimination

Systems of linear equations provide a natural starting point for Linear Algebra. The aim of this session is to develop Gaussian elimination not only as a computational algorithm, but also as a method for understanding the structure of solutions of linear systems.

The lecture will emphasize the connection between the algebraic operations performed during elimination and the solution set of the original system.

The session will also indicate how Gaussian elimination naturally leads to the notions of rank, linear independence, and dimension.

Session 2: Row Rank, Column Rank, and the Rank of a Matrix

Understanding Matrix Rank: Rows, Columns, and Linear Dependence

The notion of rank is one of the central concepts connecting matrix computations with the abstract theory of vector spaces.

Possible topics include:

An important objective of the lecture will be to move beyond the computational definition of rank.

Session 3: Vector Spaces and Subspaces

Suggested title:

From Vectors to Vector Spaces: Understanding Abstraction in Linear Algebra

This session introduces one of the first major abstractions encountered by undergraduate mathematics students.

Special attention will be given to the transition from concrete vectors in

$$\mathbb{R}^2 \quad \text{and} \quad \mathbb{R}^3$$

to abstract vector spaces.

Session 4: Linear Independence, Basis, and Dimension

Linear Independence, Basis, and Dimension

This session develops the fundamental ideas used to describe a vector space efficiently.

The relationship

$$\text{spanning} \quad + \quad \text{linear independence} \quad = \quad \text{basis}$$

will be emphasized.

We will also discuss the role of a basis as a coordinate system for an abstract vector space.

Session 5: Linear Transformations and the Rank–Nullity Theorem

Linear Transformations, Kernel, Image, and the Rank–Nullity Theorem

Linear transformations provide the natural language for studying maps between vector spaces.

For a linear transformation $T : V \longrightarrow W$, one central result is $\dim V = \dim(\ker T) + \dim(\operatorname{Im} T)$.

The lecture will emphasize that the Rank–Nullity Theorem unifies several earlier topics.

The lecture will focus how the ideas of systems of equations, matrix rank, vector spaces, dimension, and linear transformations come together naturally in the Rank-Nullity Theorem.

Part II: Real Analysis

The Real Analysis component will consist of five one-hour lectures covering sequences, continuity, and differentiation.

The lectures will emphasize the transition from the computational viewpoint of calculus to the rigorous conceptual viewpoint of Real Analysis.

Session 1: Sequences and Convergence

Sequences and Convergence: From Intuition to Definition

The notion of convergence is one of the foundational ideas of Real Analysis.

A major objective of the session is to connect the intuitive statements with the precise statements. Examples will be used to explain why each quantifier in the definition is necessary.

Session 2: Continuity

Continuity: Connecting Limits, Functions, and Local Behaviour

This session will develop continuity from both intuitive and rigorous viewpoints. The connection between sequences and continuity will be highlighted. The lecture will also compare the intuitive statement

Session 3: Properties of Continuous Functions

Fundamental Properties of Continuous Functions

The third session will focus on some of the most important consequences of continuity. The Intermediate Value Theorem will be presented as the mathematical formulation of the idea that a continuous function cannot "jump over" intermediate values.

Session 4: Differentiation

Differentiation: From Difference Quotients to Local Linear Behaviour

The purpose of this session is to revisit differentiation from the viewpoint of Real Analysis rather than merely as a collection of computational rules.

Particular emphasis will be placed on the logical relationship

$$\text{differentiability} \implies \text{continuity},$$

while the converse is false.

Session 5: Fundamental Properties of Derivatives

Suggested title:

Properties and Applications of Derivatives

The final Real Analysis session will focus on the principal structural theorems governing derivatives.

The Mean Value Theorem will be a central result of the session. The lecture will emphasize how several familiar facts follow from this theorem.

Suggested Five-Day Schedule

Day	Linear Algebra	Real Analysis
Day 1	Systems of Linear Equations and Gaussian Elimination	Sequences and Convergence
Day 2	Row Rank, Column Rank, and Matrix Rank	Continuity
Day 3	Vector Spaces and Subspaces	Properties of Continuous Functions
Day 4	Linear Independence, Basis, and Dimension	Differentiation
Day 5	Linear Transformations and the Rank–Nullity Theorem	Properties and Applications of Derivatives

Proposed Pedagogy Sessions

Session 1: From Calculation to Abstraction

From Calculation to Abstraction: Helping Students Learn Undergraduate Mathematics

One of the major difficulties faced by undergraduate students is the transition from computational mathematics to abstract mathematical thinking.

Students entering undergraduate programmes are often comfortable with questions such as solving systems of equations, calculating derivatives, or evaluating limits. However, they may find concepts such as vector spaces, linear independence, supremum, convergence, and continuity considerably more difficult.

The session will address questions such as:

- Why do students find abstract mathematical concepts difficult, and how should a new mathematical definition be introduced?
- How can teachers distinguish procedural knowledge from conceptual understanding, and how can students be encouraged to move from computation to general mathematical reasoning?

Session 2: Teaching Proof and Mathematical Reasoning: Learning Trajectory

Teaching Proof and Mathematical Reasoning at the Undergraduate Level

The transition from computational mathematics to proof-based mathematics is one of the most significant challenges faced by undergraduate students.

This session will focus on how teachers can help students read, understand, construct, and communicate mathematical proofs.

The session will address questions such as:

- What does it mean to understand a theorem, and how should students learn to unpack the statement of a theorem?
- How can students learn to construct proofs rather than memorize them and how much of a proof should a teacher reveal?

Session 3: Problem Solving and Active Learning

Learning Mathematics by Doing Mathematics: Problem Solving and Active Learning

Mathematics is learned most effectively when students actively engage with problems, examples, conjectures, and arguments.

The session will discuss ways of transforming a conventional mathematics lecture into a more participatory learning environment.

The session will address topics such as:

- Designing questions that require mathematical thinking rather than recall and think–pair–share activities in mathematics classrooms.
- Giving hints without giving away the solution and designing a sequence of problems that leads naturally to a theorem.

Session 4: Assessing Mathematical Understanding

Assessing Mathematical Understanding: Beyond Routine Examination Problems

Assessment strongly influences how students approach learning mathematics. If examinations mainly reward routine computation or memorization, students naturally concentrate on these skills.

This session will focus on designing assessments that measure mathematical understanding, reasoning, and problem-solving ability.

The session will address topics such as::

- Formative versus summative assessment and Conceptual versus computational questions.
- Designing effective tutorial sheets and Open-ended problems.
- Error analysis and grading mathematical proofs.

Session 5: Teaching a Diverse Undergraduate Mathematics Classroom

Reaching Every Student: Teaching Mathematics in a Diverse Undergraduate Classroom

Undergraduate mathematics classrooms often contain students with widely different levels of preparation, confidence, mathematical maturity, and motivation.

Some students may be strong computationally but unfamiliar with proofs, while others may struggle with basic algebra or mathematical language. At the same time, a small number of students may be considerably ahead of the rest of the class.

The session will address topics such as:

- Diagnosing gaps in prerequisite knowledge and teaching classes with widely varying mathematical backgrounds.
- Encouraging students who are hesitant to ask questions and supporting weaker students without slowing down the entire class.
- Improving mathematical communication and helping students learn how to read mathematics textbooks independently.

Appropriate mathematical technology will also be discussed in this session.

Suggested Five-Day Schedule

Day	**Pedagogy Session**	**Central Question**
Day 1	From Calculation to Abstraction	Why do students find university mathematics difficult?
Day 2	Teaching Proof and Mathematical Reasoning	How can we help students become comfortable with proofs and rigorous reasoning?
Day 3	Learning Mathematics by Doing Mathematics	How can students be made active participants in the mathematics classroom?
Day 4	Assessing Mathematical Understanding	Are our examinations testing the mathematical understanding that we want students to develop?
Day 5	Teaching the Diverse Undergraduate Classroom	How can teachers effectively address students with very different mathematical backgrounds?

Tentative Five-Day Timetable

Day	9:30-10:30	10:30-11:00	11:00-12:00	12:00-1:30	1:30-2:45	2:45-3:15	3:15-4:30
Day 1	ML 1	TB	ML 2	LB	PL 1	TB	TS
Day 2	ML 3	TB	ML 4	LB	PL 2	TB	TS
Day 3	ML 5	TB	ML 6	LB	PL 3	TB	TS
Day 4	ML 7	TB	ML 8	LB	PL 4	TB	TS
Day 5	ML 9	TB	ML 10	LB	PL 5	TB	TS

ML: Mathematics Lecture

PL: Pedagogy Lecture

TB: Tea Break

LB: Lunch Break

TS: Tutorial session