Density functions for the epsilon multiplicity of homogeneous ideals

Suprajo Das

Abstract

Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension d, and let $I \subseteq R$ be an ideal. Ulrich and Validashti defined the *ε-multiplicity* of I as

$$\varepsilon(I) = \limsup_{n \to \infty} \frac{\lambda_R(H^0_{\mathfrak{m}}(R/I^n))}{n^d/d!}.$$

This invariant may be viewed as a generalization of the classical Hilbert-Samuel multiplicity. Cutkosky showed that the lim sup in this definition can be replaced by an actual limit when R is analytically unramified. A surprising example due to Cutkosky, Hà, Srinivasan, and Theodorescu demonstrates that this limit can be an irrational number even when R is a regular local ring.

In this talk, we shall focus on the case of homogeneous ideals in a standard graded domain over a field. Motivated by Trivedi's approach to the Hilbert-Kunz multiplicity via density functions, we introduce a compactly supported continuous real-valued function, called the *ε-density function*, whose integral recovers the ε-multiplicity. If time permits, we will present explicit examples and discuss applications in the context of integral closures.

This talk is based on joint works with Roy and Trivedi.