Divided difference with repetitions

02 April 2026 12:43

Divided difference with repetitions.

Recall that if point $x_0, x_1, \dots, x_n$ are distinct, then $f[x_0, x_1, \dots, x_n]$ has been defined as the coefficient of x^n in the polynomial from Π_n that interpolates f at $x_0, x_1, \dots, x_n$.

We need to define the meaning of interpolation when the list of points contains repetitions.

Definition. We say that f interpolates 0 at $x_1, \dots, x_n$ if $f^{(k-1)}(\xi) = 0$ for each point ξ that occurs k in the list $x_0, x_1, \dots, x_n$.

Example. We say that f interpolates 0 at $1, 3, 8, 1, 13, 1, 8$ if

$$f(1) = f'(1) = f''(1) = 0$$
$$f(8) = f'(8) = 0$$
$$f(13) = 0. \qquad f(3) = 0.$$

It is easy to verify that (Exc.) a polynomial p interpolates 0 at $x_0, x_1, \dots, x_n$ iff $p(x)$ contains the factor

$$q(x) = \prod_{j=0}^{n} (x - x_j).$$