

Numerical differentiation - Difference formulae

16 April 2026 12:24

Numerical differentiation

Suppose that values of a function f are given at points x_0, x_1, \dots, x_n .
Can we get an estimate of $f'(x_i)$?

Forward difference formula:

$$D_+ f(x) = \frac{f(x+h) - f(x)}{h}$$

Backward difference formula:

$$D_- f(x) = \frac{f(x) - f(x-h)}{h}$$

Central difference formula:

$$D_0 f(x) = \frac{f(x+h) - f(x-h)}{2h}$$

Suppose that f and f' are continuous on the closed interval $[x, x+h]$ and f'' exists on $(x, x+h)$. A Taylor's Theorem shows that

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(\xi)$$

for some ξ between x and $x+h$.

$$\Rightarrow \frac{f(x+h) - f(x)}{h} = f'(x) + \frac{h}{2} f''(\xi)$$

$$\Rightarrow |D_+ f(x) - f'(x)| \leq \frac{h}{2} \|f''\|_{L^\infty(x, x+h)}$$

Exc. Carry out the same exercise for $D_- f(x)$.