Regula Falsi

10 February 2026 12:23

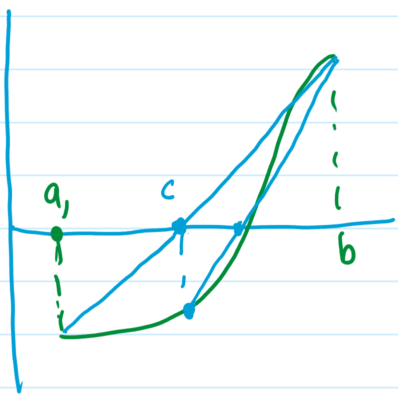
Regula Falsi or Method of false position

Bisection method has the advantage that if $f(a)f(b) < 0$, then the iterative sequence converge to a root. However, this convergence is slow under the circumstances

a) if the interval $[a,b]$ is large

b) if the root is near one of the end points.

In regula falsi method, we approximate the function $y = f(x)$, by the straight line joining $(a, f(a))$ and $(b, f(b))$ and the root α is approximated by the x-intercept of this line.



Equation of this line is given by

$$y = f(a) + \frac{f(b) - f(a)}{b-a}(x-a)$$

The x-intercept is given by (denoted by x_1)

$$f(a) + \frac{f(b) - f(a)}{b-a}(x_1 - a) = 0$$

$$\Rightarrow x_1 = \frac{af(b) - bf(a)}{b-a}$$

Now we proceed as in the bisection method:

If $f(x_1) = 0$, then the root $\alpha = x_1$

If $f(a) \cdot f(x_1) < 0$, then $[a_1, b_1] = [a, x_1]$ and repeat

If $f(a) f(x_1) > 0$, then $[a_1, b_1] = [x_1, b]$ and repeat

We generalise the procedure as follows:

1. For $n = 0, 1, 2, \cdots$ define the iterative sequence by

$$x_{n+1} = a_n - f(b_n)\frac{b_n - a_n}{f(b_n) - f(a_n)} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$$

which is the x-intercept of the line joining $(a_n, f(a_n))$, $(b_n, f(b_n))$

2. If x_{n+1} solves $f(x_{n+1}) = 0$, then that is the root.

3. Define the subinterval $[a_{n+1}, b_{n+1}]$ of $[a_n, b_n]$ as follows:

$$[a_{n+1}, b_{n+1}] = \begin{cases} [a_n, x_{n+1}] & \text{if } f(a_n) f(x_{n+1}) < 0 \\ [x_{n+1}, b_n] & \text{if } f(b_n) f(x_{n+1}) < 0 \end{cases}$$

Bisection method always converge as the length of the intervals always go to zero. However, the length of subintervals in RF method may not go to zero if f is concave or convex in the interval $[a, b]$. Therefore, we cannot specify a stopping criteria in the case of RF method

Convergence analysis of RF method

We obtain two sequences from regula falsi method.

1. $a \le a_1 \le a_2 \le \cdots \le b$ (bdd above sequence monotonically non-decreasing)

Hence $\lim_{n\to\infty} a_n = \alpha$ exists.

2. $a \le \cdots \le b_1 \le b_0 \le b$ (bdd below sequence monotonically non-increasing)

Hence $\lim_{n\to\infty} b_n = \beta$ exists.

Since $a_n \le b_n$, we can also conclude that

$$\alpha = \lim_{n\to\infty} a_n \le \lim_{n\to\infty} b_n = \beta.$$